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ScienceDirect
Journal of Sound and Vibration 301 (2007) 126–145
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Finite element modelling and updating of a lively footbridge: The complete process

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Received 27 July 2005; received in revised form 4 May 2006; accepted 18 September 2006
Available online 14 November 2006

Abstract

The finite element (FE) model updating technology was originally developed in the aerospace and mechanical engineering disciplines to automatically update numerical models of structures to match their experimentally measured counterparts. The process of updating identifies the drawbacks in the FE modelling and the updated FE model could be used to produce more reliable results in further dynamic analysis. In the last decade, the updating technology has been introduced into civil structural engineering. It can serve as an advanced tool for getting reliable modal properties of large structures. The updating process has four key phases: initial FE modelling, modal testing, manual model tuning and automatic updating (conducted using specialist software). However, the published literature does not connect well these phases, although this is crucial when implementing the updating technology. This paper therefore aims to clarify the importance of this linking and to describe the complete model updating process as applicable in civil structural engineering. The complete process consisting the four phases is outlined and brief theory is presented as appropriate. Then, the procedure is implemented on a lively steel box girder footbridge. It was found that even a very detailed initial FE model underestimated the natural frequencies of all seven experimentally identified modes of vibration, with the maximum error being almost 30%. Manual FE model tuning by trial and error found that flexible supports in the longitudinal direction should be introduced at the girder ends to improve correlation between the measured and FE-calculated modes. This significantly reduced the maximum frequency error to only 4%. It was demonstrated that only then could the FE model be automatically updated in a meaningful way. The automatic updating was successfully conducted by updating 22 uncertain structural parameters. Finally, a physical interpretation of all parameter changes is discussed. This interpretation is often missing in the published literature. It was found that the composite slabs were less stiff than originally assumed and that the asphalt layer contributed considerably to the deck stiffness.

1. Introduction

As civil engineering structures, and in particular footbridges, are becoming increasingly slender due to improvements in construction materials and technology, they are also becoming lighter and less damped. In principle, this means that new footbridge structures tend to be easier to excite than older ones and there is

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0022-460X/$ - see front matter
doi:10.1016/j.jsv.2006.09.024

a higher potential for vibration serviceability problems to occur. This has already been experienced by many new structures in the last decade—the new London Millennium Bridge [1] being a particularly high-profile example. For slender and lightly damped bridges, their dynamic response due to near-resonant excitation governs their vibration performance. When doing response calculations in design, simulation of this type of near-resonant dynamic response is very sensitive to even small variations in modal properties, such as damping ratio, natural frequency and modal mass. These are key input parameters in the analysis. Therefore, knowing modal properties of a footbridge, together with its mode shapes, as precisely as possible has become very important. This is important not only for the design of new structures with similar layouts, but also for the rectification of existing lively footbridges, as well as for seismic analysis and general research into vibration serviceability. However, despite the huge importance of modal properties in the assessment of vibration performance of footbridges, their reliability when predicted via finite element (FE) modelling is still rather uncertain. The main reason for this is the general lack of information on modal properties of as-built footbridge structures and their correlation with FE modelling based on design data and best engineering judgement.

Developing a numerical model of a civil engineering structure that has sufficiently reliable dynamic properties is a complex issue. It requires a rather wide range of skills and expertise in areas as diverse as FE modelling, modal testing of full-scale structures and FE model correlation, tuning and updating with the regard to experimentally obtained modal properties. This methodology is nowadays used routinely in the mechanical and aerospace engineering disciplines, where prototyping is part of a normal design process of structures subject to dynamic loading.

Unfortunately, prototyping is not common in civil structural engineering design. Therefore, all this cannot be done easily during the design (of, say, a footbridge) bearing in mind that the modal testing can be conducted only on an already built structure, which is a unique 'prototype' never to be built again. Thus, it may appear that the whole idea of getting reliable structural modal properties by FE modelling, modal testing, and FE model correlation and updating is pointless in the case of civil engineering structures after they are built. However, this is not the case as exercises like these are the only reliable way to gauge our ability to predict vibration behaviour of future civil engineering structures. The whole process of FE modelling, modal testing, and FE model correlation and updating adds to the currently very limited body of knowledge on vibration performance of as-built structures with significant potential to use this knowledge in future designs.

Therefore, the aim of this paper is to demonstrate the complete combined analytical and experimental process required to obtain as reliable as possible estimates of modal properties of a steel box girder footbridge. For this purpose, every phase of the process and its purpose will be first explained briefly, with particular attention paid to the automatic FE model updating procedure, which is a new technology still not used commonly in civil engineering.

However, in current civil structural engineering design practice, it has become common to develop an FE model of the structure and use it for calculation of its static and/or dynamic responses. To obtain a good model, it is necessary to reduce the mathematical modelling errors to an acceptable level. Therefore, the assumptions on which the model is based should be evaluated carefully. Nevertheless, even with most careful and detailed numerical modelling based on design data available and best engineering judgement, differences regularly occur between the modal properties of an as-built structure and their counterparts predicted numerically. This is typically due to inevitable uncertainties linked with modelling of, in the case of footbridges, boundary conditions, material properties, and effects of non-structural elements, such as handrails and asphalt [2].

It should be stressed here that the errors in the natural frequencies for footbridges predicted by very reasonable FE model in the design can be as large as 37% [3]. Not surprisingly, it is now widely accepted that modal testing and modal properties estimated from it are much more reliable than FE modelling for assessing dynamic performance of as-built structures [4,5].

Once the modal dynamic properties of a footbridge (mainly natural frequencies and mode shapes) are identified experimentally and the level of error introduced by the initially developed FE models is identified, their drawbacks in the FE modelling can be found and the initial FE model can be corrected. This procedure is called FE model updating, and can be considered as an attempt to use the best features from both the experimental and analytical model [5]. The former gives more reliable modal properties of the structure, including modal damping which cannot be obtained analytically, while the latter retains very detailed representation of the structure.

In this paper, first presented is a background review regarding the FE model updating technology. This is followed by examples of implementation of the procedure in civil structural engineering, especially in bridge engineering. After this, a lively steel box girder footbridge is described and the initial FE model is presented. After the presentation of experimentally identified modal properties, the manual tuning necessary to prepare the FE model for an automatic updating is given. Then, a sensitivity-based automatic model updating is conducted and results are discussed.

2. Background review

In this section general information about FE model updating techniques is given first, followed by their implementation in civil structural engineering.

2.1. Finite element model updating

The FE model updating procedure typically minimises the differences between the FE and experimentally estimated modal properties. This is done by changing some uncertain FE modelling parameters, which have the potential to influence modal properties. The resulting FE model can then be used in further analyses.

The updating process typically consists of manual tuning and then automatic (or formal) model updating using some specialised software. The manual tuning involves manual changes of the model geometry and modelling parameters by trial and error, guided by engineering judgement. The aim of this is to bring the numerical model closer to the experimental one. Often, in this process an analyst is able to improve the initial structural idealisation typically related to boundary conditions and non-structural elements. This process usually includes only a small number of key parameters manageable manually. The aim of automatic updating is to improve further the correlation between the numerical and experimental modal properties by taking into account a larger number of uncertain parameters.

The term ‘parameter’ is used here for all input values which define the numerical model. Moreover, all measured modal properties which are targeted in the updating process will be called ‘target responses’ hereafter.

It is important to emphasise here that not all FE models of a structure can be updated. To have a successful automatic updating of an FE model, it is necessary to prepare the initial FE model for it. To do this, firstly it would be necessary to minimise discretisation errors and to use modelling strategies which can represent truly all important aspects of structural behaviour and geometry [6,7]. This means that careful attention should be paid to model geometry and various other details. This is important because the automatic model updating procedure cannot easily correct large errors in the geometry of the initial modelling. It can only rectify the errors caused by uncertainties of modelling parameters in a geometrically well-defined model. Also, when preparing the FE model for the automatic updating, the differences between analytical and experimental modal responses (usually natural frequencies and mode shapes) should be as small as practicable. If they are too large, the automatic updating procedure can have numerical difficulties and/or produce physically unrealistic parameter changes during the updating process. These are reasons to recommend the manual tuning (by trial and error and engineering judgement) of the initial FE model first. The tuned model should therefore feature meaningful starting parameters for the formal updating [8,9].

Formal FE model updating is now a mature technology. It is widely used in the mechanical and aerospace engineering disciplines to update analytical models of structural prototypes. A large number of updating procedures exist [4,10] and their detailed discussion is beyond the scope of this paper. Here, only the principles on which model updating is based are reviewed briefly.

2.2. Basic theory used in FE model updating

The main idea in formal FE model updating of minimising the differences between the analytical and experimental models is, in essence, an optimisation problem. This problem can be solved in many different ways. In general, there are two groups of updating methods: direct methods and iterative (or parametric) methods. The former are based on updating of stiffness and mass matrices directly, in a way that often has no physical meaning. The latter, on the other hand, concentrates on the direct updating of physical parameters which indirectly update the stiffness and mass matrices in which these parameters feature [5,11]. Iterative methods are slower than their direct counterparts. However, their main advantage is that changes in the updated model can be interpreted physically. Also, iterative methods can be implemented easily using existing FE codes [12]. These are the main reasons why iterative methods are widely accepted and now used almost exclusively in the updating exercises. This link between the iterative updating and the physical world is very important in civil structural engineering and is the main reason why only this type of updating is considered in this paper. Numerous examples of implementation of direct methods, mainly in mechanical engineering and control theory, can be found elsewhere [10,13,14].

The iterative methods are mainly sensitivity based. This requires the sensitivity matrix **S** to be calculated in every iteration. The sensitivity matrix is a rectangular matrix of order $m \times n$, where m and n are the number of target responses and parameters, respectively [15]:

$$\mathbf{S} = [S_{ij}] = \left[\frac{\delta R_i}{\delta P_j}\right]. \tag{1}$$

S_{ij} is the sensitivity of the target response R_i ($i = 1,2,\ldots,m$) to a certain change in parameter P_j ($j = 1,2,\ldots,n$). Operator δ presents the variation of the variable. Elements of the sensitivity matrix can be calculated numerically using, for example, the forward finite difference approach [15]:

$$S_{ij} = \frac{R_i(P_j + \Delta P_j) - R_i(P_j)}{(P_j + \Delta P_j) - P_j}, \tag{2}$$

where $R_i(P_j)$ is the value of the ith response at the current state of the parameter P_j, while $R_i(P_j+\Delta P_j)$ is the value of the same ith response when the parameter P_j is increased by value ΔP_j.

Obviously, for calculation of the sensitivities, the relevant target responses and structural parameters should be selected. The target responses should be chosen between those measured. The responses which are mainly considered in civil engineering applications are natural frequencies, mode shapes and frequency response functions (FRFs), or some combination of these. The choice depends on the measured data available, their quality, and (non)existence of close modes [4]. As a rule, only high-quality measured modal properties should be used as target responses. As natural frequencies are normally measured quite accurately, they are almost always selected. If close modes are present, FRFs might be a better choice for target responses.

Selection of updating parameters is probably the most important step on which the success of the model updating depends. It is recommended to choose uncertain parameters only, and between them to choose those to which the selected target responses are most sensitive. Also, the number of parameters should be kept to an absolute minimum. All this is to avoid numerical problems due to ill-conditioning [11].

Once relevant (measured) target responses and structural parameters for updating have been selected, the sensitivity matrix can be calculated. Since in the iterative model updating process the updating parameters change at every step, the sensitivity matrix has to be recalculated in each iteration. Let us denote, for a given iteration, the starting parameter and target response vectors as $\mathbf{P}_0$ and $\mathbf{R}_0$, respectively. The vector of updated parameters in the current iteration is $\mathbf{P}_u$, while the target response vector obtained experimentally is $\mathbf{R}_e$. The targeted experimental response vector $\mathbf{R}_e$ can be approximated via vectors $\mathbf{R}_0$, $\mathbf{R}_u$ and $\mathbf{R}_0$ using the linear term in a Taylor's expansion series:

$$\mathbf{R}_e \approx \mathbf{R}_0 + \mathbf{S}(\mathbf{P}_u - \mathbf{P}_0). \tag{3}$$

The iterative process is required here because the relationship between target responses and parameters that is mainly nonlinear is approximated by the linear term. This means that updating parameters need to be changed by a small amount in each iterative step until the required minimum difference between the calculated and experimentally measured responses is achieved. Therefore, the finally updated parameters cannot be calculated in a single step [16].

The task of updating aimed at finding parameter values $\mathbf{P}_u$ in the current iteration can be solved in different ways such as using a pseudo-inverse (least squares) method, weighted least squares or Bayesian method. This depends on whether weighting coefficients for parameters and/or target responses are used as is the case in last two methods [12]. The purpose of these weighting coefficients is to give different significance to numerical parameters and measured target responses depending on the confidence in these data. For example, weighting coefficients for responses take into account the confidence in the measured values, which is typically higher for natural frequencies than for mode shapes. Weighting coefficients for input parameters take into account the degree of uncertainty in them. The more uncertain a parameter is, the lower is the confidence in it, which means that the weighting value is lower too.

If a Bayesian method is chosen, which is often the case in the commercially available model updating software, then the aim of the updating procedure is not to simply minimise the difference between numerical and measured target responses. Instead, an error function which includes differences, not only between the target experimental and numerical responses, but also between updating parameters in two successive iterations as well as parameters and target responses' weights, is defined. In this way the aim of the updating procedure is to minimise response differences $\Delta\mathbf{R}$ and simultaneously to ensure convergence of the process via minimisation of parameter differences $\Delta\mathbf{P}$ in two successive iterations. Therefore, this error function is, in general, defined as a function of input parameters and target responses, as well as the weighting factors. The error function used for the case study presented in this paper is defined as [15]

$$E(\Delta\mathbf{R},\ \Delta\mathbf{P},\ \mathbf{C}_R,\ \mathbf{C}_P) = \Delta\mathbf{R}^{\mathrm{T}} \cdot \mathbf{C}_R \cdot \Delta\mathbf{R} + \Delta\mathbf{P}^{\mathrm{T}} \cdot \mathbf{C}_P \cdot \Delta\mathbf{P}, \tag{4}$$

where $\Delta\mathbf{R} = \mathbf{R}_e - \mathbf{R}_0$ is the vector which represents the errors in target responses while $\Delta\mathbf{P} = \mathbf{P}_u - \mathbf{P}_0$ is the vector of parameter changes. $\mathbf{C}_R$ and $\mathbf{C}_P$ are diagonal matrices of weighting coefficients for target responses and parameters, respectively, and both should be defined by the analyst based on their experience. Higher values of these coefficients indicate greater confidence. From Eq. (4) it can be seen that the greater the confidence, the finer tuning of the corresponding quantities is needed to make the error sufficiently small. On the other hand, the parameters and target responses in which the confidence is small will not contribute significantly to the error value and therefore will have a less strong influence on the final results.

Using the linear relationship between the target responses and parameters given in Eq. (3), estimating the confidence into the parameters and target responses and expressing parameter differences $\Delta\mathbf{P}$ in the current iteration as

$$\Delta\mathbf{P} = \mathbf{P}_u - \mathbf{P}_0 = \mathbf{G}(\mathbf{R}_e - \mathbf{R}_0) \tag{5}$$

matrix $\mathbf{G}$ can be found in the way to minimise the error function [16,17]. It can be proven that matrix $\mathbf{G}$ in the case when there are more responses than parameters ($m>n$) is [15]

$$\mathbf{G} = (\mathbf{C}_P + \mathbf{S}^{\mathrm{T}}\mathbf{C}_R\mathbf{S})^{-1}\mathbf{S}^{\mathrm{T}}\mathbf{C}_R. \tag{6}$$

Otherwise, when there are more parameters than responses ($n>m$) matrix $\mathbf{G}$ is

$$\mathbf{G} = \mathbf{C}_P^{-1}\mathbf{S}^{\mathrm{T}}(\mathbf{C}_R^{-1} + \mathbf{S}\mathbf{C}_P^{-1}\mathbf{S}^{\mathrm{T}})^{-1}. \tag{7}$$

Bearing all this in mind, the updating procedure can be summarised as follows:

1. Choose the weighting factors for parameters and target responses.
2. Calculate the sensitivity matrix $\mathbf{S}$ for the given state of parameters $\mathbf{P}_0$ and responses $\mathbf{R}_0$.
3. Calculate matrix $\mathbf{G}$ using either Eqs. (6) or (7).
4. Using experimental response vector $\mathbf{R}_e$, the updated parameter vector $\mathbf{P}_u$ can be obtained via a re-arranged Eq. (5):

$$\mathbf{P}_u = \mathbf{P}_0 + \mathbf{G}(\mathbf{R}_e - \mathbf{R}_0). \tag{8}$$

5. The new response vector which corresponds to updated parameters P_u should then be calculated as a result of modal analysis. This response vector and the vector of updated parameters then become the starting vectors $\mathbf{R}_0$ and $\mathbf{P}_0$ for the next iteration.

The procedure then goes back to step 2 to calculate a new sensitivity matrix (which changes whenever the model is updated between two iterations). Steps 2–5 are repeated until a satisfactory convergence of numerical responses to the experimental data is achieved (that is until the error function is minimised to a prespecified tolerance).

An updating process which produces good correlation between experimental and analytical responses can be regarded as successful only if finally obtained parameters are physically viable. If not, then either a different error function or different parameter selection, or both, should be considered [11]. Also, some changes in the weighting matrices should be considered, having in mind that these coefficients can be difficult to guess correctly first time round [16]. Therefore, it is expected that some kind of additional trial and error approach is used before a satisfactory set of updated parameters is obtained.

Generally, the updating which targets larger number of measured responses at a time is preferable because it puts more constraints to the optimisation process. Successful updating in this case becomes more difficult but once it is achieved it gives more confidence in the results than the same procedure using only a few responses. This becomes clear if a simple example is considered with only one target response, say a natural frequency. There is an infinite number of ways to achieve good correlation for this response by changing either only one parameter at a time or some combination of them. In this way, it is not possible to decide which parameter change is most realistic. Therefore, targeting more responses at a time decreases the number of combinations for parameter changes. Finally, to ensure that parameter changes are physically possible, some additional constraints in the form of physically acceptable limits for updating parameters can also be introduced. This makes sure that if the parameter reaches its limit in a particular iteration, it will stay constant through all subsequent iterations until the end of the updating process.

Finally, the success of the updating process is usually judged through a comparison of natural frequencies, overlaying mode shapes and calculation of the modal assurance criterion (MAC) and the coordinate modal assurance criterion (COMAC). However, if the measured responses are not particularly reliable (say from noisy data), then convergence of the iterative procedure can become a problem. It seems that higher modes are more difficult to update in this situation [9]. Also, if a measurement grid is not dense enough to prevent spatial aliasing, the MAC values can suggest correlation between modes which are otherwise linearly independent [18].

2.3. Applications in civil structural engineering

Over the last decade, there have been several attempts to transfer the updating technology from the mechanical and aerospace engineering to civil structural engineering. The whole procedure is more difficult to implement in civil engineering because of the larger size of the structures leading to poorer quality of experimental data gathered in open-space noisy environments. Also, the inherent nonlinear amplitude dependant behaviour, the presence of numerous non-structural elements and difficult to define boundary conditions mean that the structural modelling parameters are not so controllable as is often the case in the mechanical and aerospace disciplines. However, some successful examples of updating in civil engineering do exist and are presented here.

In principle, papers dealing with the complete process of experimental modal testing and analytical/numerical modelling and updating of civil engineering structures are rare. However, there are many good papers devoted to modal testing of civil engineering structures [19–24]. As the modal testing technology has developed and been accepted as a way for reliable estimation of dynamic properties, more researchers have started to pay attention to the correlation between the initial FE model and experimental results from real-life as-built structures. In this process, the structural parameters which influenced the analytical results most and managed to shift them towards the experimental ones were identified in general. In the case of footbridges these are stiffness of supports and non-structural elements (decks, asphalt surfacing and handrails) as well as material properties, such as dynamic modulus of elasticity for concrete [25–27].

The logical step forward was then to try the automatic updating procedure by using specialist software developed for this purpose. The procedure was successfully implemented on different types of structure, such as a 48-storey building [28], a high rise tower [12], road and/or rail bridges [9,29,30] and two footbridges [8]. Also, model updating has been attempted as a tool for damage identification [6,31,32].

Reviewed papers suggest that the automatic updating of full-scale road and railway bridges might have difficulties in achieving a high level of correlation with experimental results. For example, when updating a 750 m long road and railway bridge, Zhang et al. [29] got a maximum frequency error in the updated model of about 10%. A similar result was obtained by Brownjohn and Xia [9] for a curved road bridge spanning 100 m. Maximum frequency difference for a 90 m long road bridge of 6.2% was obtained by Jaishi and Ren [30]. On the other hand, the automatic updating conducted by Pavic et al. [8] for two footbridges spanning 34 and 20 m produced maximum frequency difference of only 2.0% and 1.1%, respectively. It, therefore, seems that it is easier to update smaller bridges, such as pedestrian ones. This is not surprising considering that larger structures tend to have many more features which are important for their dynamic behaviour (e.g. connections, supports, etc.) but are difficult to model in detail in the FE model. Also, experimental data on larger structures tend to be of poorer quality compared with their smaller counterparts. Moreover, it is worth noting that, for example, Zhang et al. [29] conducted updating which targeted as many as 17 measured natural frequencies, which put lot of constraints to the optimisation procedure, whilst in the case of the footbridge where the maximum frequency error was 1.1% the updating was done according to natural frequencies and mode shapes for three measured modes only [8]. Regarding MAC values, in most cases, they were higher than 0.80, which is a very good mode shape agreement for civil structural engineering applications of updating.

3. Description of test footbridge structure

The investigated footbridge spans 104 m over the Morača River in Podgorica, capital of Montenegro (Fig. 1). The structural system of the Podgorica footbridge is a steel box girder with inclined supports. The structure's main span between inclined columns is 78 m and it has two side spans of 13 m each. The top flange of the main girder forms a 3 m wide deck. The depth of the girder varies from 1.4 m in the middle of the central span to 2.8 m at the points where the inclined columns connect to the main box girder (Fig. 2). Along its whole length the box girder is stiffened by longitudinal and transverse stiffeners, as shown in Fig. 2. The connection between the inclined columns and box girder is strengthened by vertical stiffeners visible in Fig. 1. Water supply and drainage pipes pass through the steel box section (Fig. 2), which are suspended from the top flange of the main girder.

After its construction in the early 1970s, the footbridge fundamental natural frequency for the vertical mode was in the region of the normal walking frequencies, which is 1.5–2.4 Hz [33]. This was the reason for the bridge to experience strong vibrations in the vertical direction under pedestrian walking excitation. Additionally, a high concentration of stresses under a particular static load combination was found by

Fig. 1. Photograph of the Podgorica footbridge.

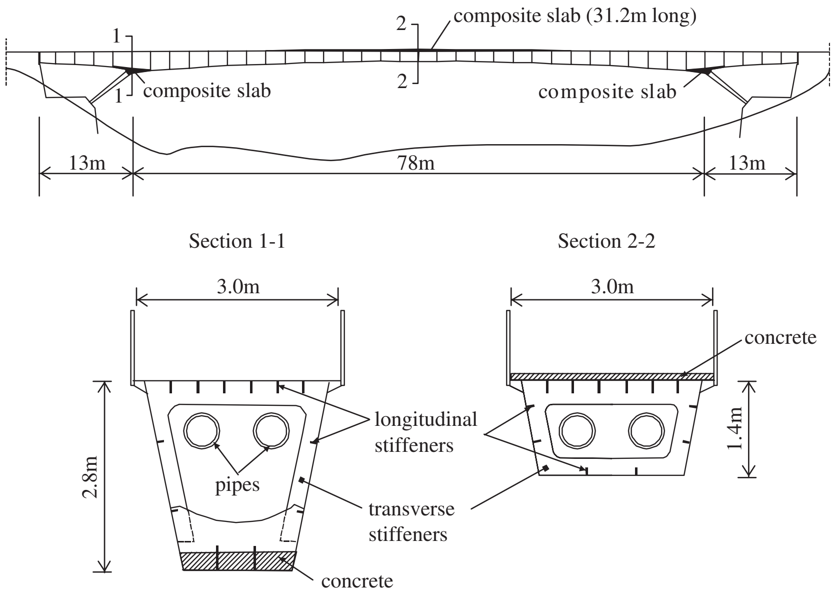

Fig. 2. General arrangement drawing (not to scale) of Podgorica footbridge.

a calculation not performed during design. Consequently, the footbridge was strengthened by a concrete slab cast over the bottom steel flange in the regions around the columns as well as over the top flange of the box girder in the central part of the main span (Fig. 2). At the same time, additional steel plates were added to the box columns. However, all this added not only stiffness but also some mass to the dynamic system. Consequently, the fundamental natural frequency did not change very much and the footbridge still remains very lively.

4. Initial FE modelling, modal testing and model tuning

This section describes the development of the initial FE model for the footbridge investigated as well as its modal testing. Then, the manual model tuning is discussed.

4.1. Initial finite element modelling

To minimise discretisation and modelling errors a very detailed initial 3D FE model was developed using the ANSYS FE code [34]. This initial FE model is described in detail by Živanović et al. [35]. The only difference between that model and the starting FE model used in this paper is that the late information about additional steel plates used to strengthen the columns in the rectification phase is now taken into account. The model is shown in Fig. 3 and will be described briefly in this section.

The main steel box girder and its longitudinal and transverse stiffeners and box section columns were modelled using orthotropic SHELL63 elements assuming isotropic properties. These elements are capable of transferring both in-plane and out-of-plane loads. In the absence of more precise data, it was assumed that two different plates used for strengthening of columns were as thick as the types of plates used in the initial design, that is 2 and 3 cm. The composite steel–concrete slabs at three locations on the bridge structure (Fig. 2) were modelled using an equivalent steel thickness and, again, SHELL63 elements with isotropic property. The water and drainage pipes were modelled as distributed mass along the lines connecting points at which the pipes were suspended from the bridge deck. The mass was calculated by assuming that water filled a half of the pipes' volume. The handrails were modelled using 3D BEAM4 elements while inclined column supports were modelled as fully fixed considering solid rock foundations. Supports at both ends of the main girder were modelled as pinned, but with a possibility to slide free in the longitudinal direction (Fig. 3: inset).

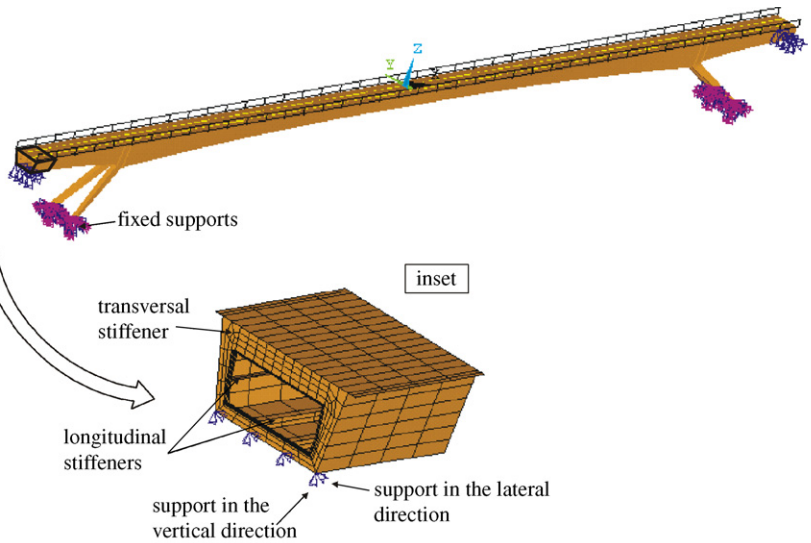

Fig. 3. Initial FE model.

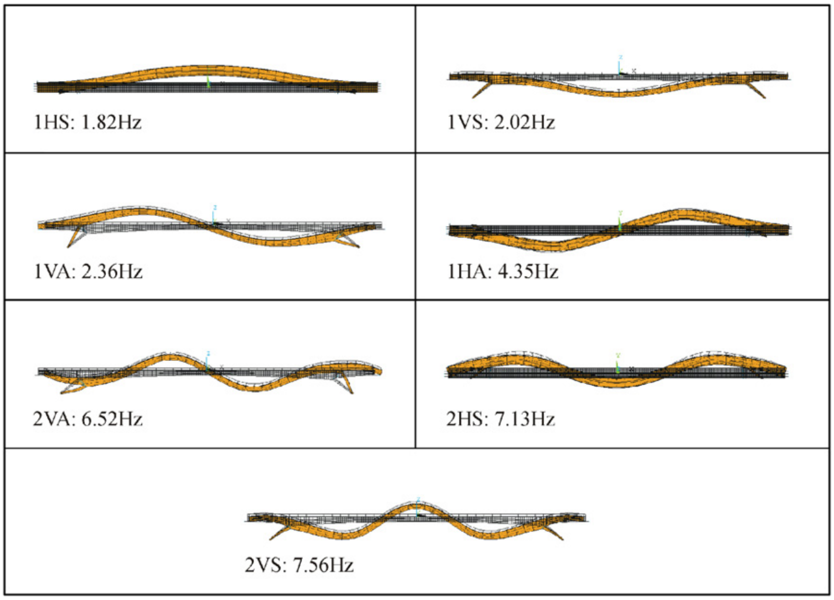

Fig. 4. Modes of vibration calculated from the initial FE model.

Seven lowest modes of vibration from the initial FE model are presented in Fig. 4. Labels H and V stand for the horizontal and vertical modes, respectively. Similarly, S and A stand for the symmetric and anti-symmetric modes, respectively.

4.2. Modal testing

Modal testing based on FRF measurements was conducted to verify the seven lowest modes of vibration obtained in the initial FE model (Fig. 4). First, modal testing for identification of the vertical modes was conducted. After this, the testing was repeated for the horizontal modes. During these measurements, the footbridge was closed for pedestrian traffic.

The excitation source for the FRF-based testing was an electrodynamic shaker that generated a chirp excitation signal. Its frequency range was chosen to be 1–9 Hz based on the expected frequencies of the modes obtained in the initial FEM. The shaker was placed at a quarter point of the main span between inclined columns (Fig. 2) since this test point is expected to respond in all vibration modes of interest. The dynamic force induced by the shaker was measured by a piezoelectric accelerometer attached to its armature. The same type of transducer was used for the structural response measurements at seven points along the bridge.

The measurement procedure is described in detail in a previous paper [35]. Here, we will only add that the identification of modal properties was conducted by using a MIMO parameter estimation procedure available in the ICATS software [36]. The estimated natural frequencies and corresponding damping ratios are presented in columns II and III in Table 1.

4.3. FE model tuning

All seven modes identified experimentally had their counterparts in the initial FE model (Table 1, column V). However, the sequence of 2VA and 2HS FE modes was reversed compared to their experimental counterparts (Table 1: columns I and IV). Also, natural frequencies of all experimental modes were underestimated, with the frequency error being exceptionally high (29.8%) for mode 1VA (Table 1: column VI). Another mode with quite large error of 13.8% was also vertical and anti-symmetric one (2VA). On the other hand, all mode shapes were well correlated, with the minimum MAC being 0.81. Something was clearly wrong with the prediction of anti-symmetric modes in the initial FE model. After visually inspecting modes, it was established that the key difference between the vertical symmetric and anti-symmetric modes was the horizontal longitudinal motion of the deck ends. As this motion was allowed, it was much more pronounced in the case of anti-symmetric modes. Therefore, adding stiffness which would be engaged by this motion would affect anti-symmetric modes much more than the symmetric ones.

Indeed, a parametric study revealed that introducing the horizontal springs in the longitudinal direction at girder ends instead of free edges could improve significantly the correlation between measured and analytical vertical modes, in particular the anti-symmetric ones [35]. The stiffness of these springs (modelled as COMBIN14 element in ANSYS) was varied by trial and error until the best correlation with measured frequencies was obtained. A stiffness value of 100 MN/m per metre width of the bridge deck produced the smallest difference between the measured and FE-calculated natural frequencies for the first four vertical modes of vibration (Fig. 5). This value was adopted in the manually tuned FE model developed prior to automatic updating. Also, in this way the sequence of mode appearance became the same as in the experimental model, and frequency error was decreased significantly with the maximum value being 4.0% for mode 1HA (Table 2). The MAC values were improved only slightly.

The data given in Tables 1 and 2 are also presented graphically in Fig. 6. The ratio between analytical and measured natural frequencies for the seven modes of vibration is given for the initial FE model and the

Table 1
Correlation between experimental and initial FE model

I Exp. mode #	II Modal testing f (Hz)	III ζ (%)	IV FE mode #	V Initial FEM f (Hz)	VI Difference $(f_V-f_{II})/f_{II}$ (%)	VII Mode shape correlation MAC (%)
1	1.83 (1HS)	0.26	1	1.82 (1HS)	−0.6	99.5
2	2.04 (1VS)	0.22	2	2.02 (1VS)	−1.0	99.7
3	3.36 (1VA)	1.86	3	2.36 (1VA)	−29.8	97.8
4	4.54 (1HA)	0.98	4	4.35 (1HA)	−4.2	98.6
5	7.35 (2HS)	2.68	6	7.13 (2HS)	−3.0	80.7
6	7.56 (2VA)	0.76	5	6.52 (2VA)	−13.8	86.6
7	7.98 (2VS)	0.60	7	7.56 (2VS)	−5.3	97.9

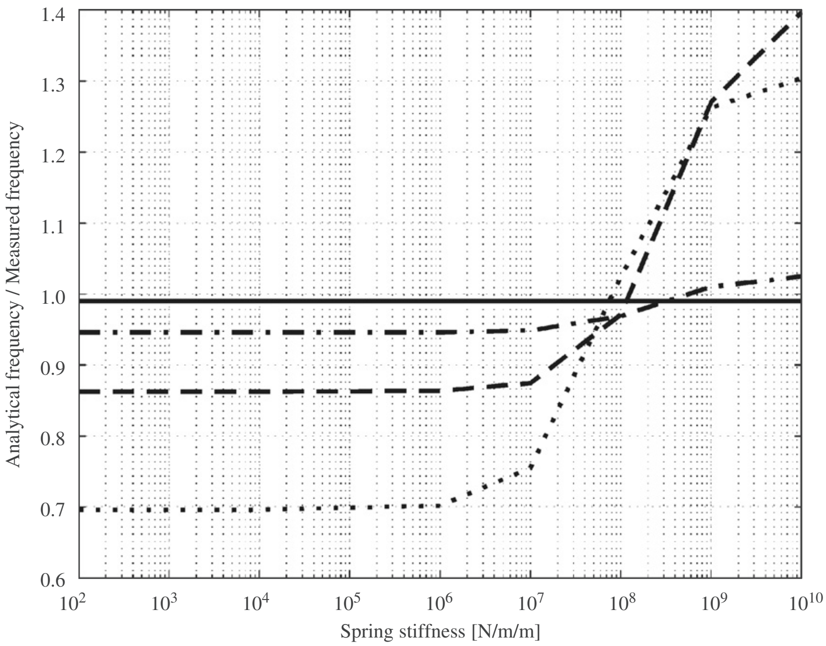

Fig. 5. Choice of appropriate stiffness for spring supports at the ends of the box girder (1VS—solid line; 1VA—dotted line; 2VA—dashed line; 2VS—dot–dashed line).

Table 2
Correlation between experimental and manually tuned FE model

I Exp. mode #	II Exp. model f (Hz)	III Tuned model f (Hz)	IV Frequency error $(f_{III}-f_{II})/f_{II}$ (%)	V Mode shape correlation MAC (%)
1	1.83 (1HS)	1.82 (1HS)	−0.6	99.5
2	2.04 (1VS)	2.02 (1VS)	−1.0	99.7
3	3.36 (1VA)	3.47 (1VA)	3.3	99.9
4	4.54 (1HA)	4.36 (1HA)	−4.0	98.7
5	7.35 (2HS)	7.15 (2HS)	−2.7	81.1
6	7.56 (2VA)	7.34 (2VA)	−2.9	88.9
7	7.98 (2VS)	7.74 (2VS)	−3.0	98.0

manually tuned model. Having in mind that the information about column strengthening was not present in the original design data available, and that this information was found some time after the first FE model had been developed [35], it is also interesting to show the frequency error which would have resulted from not introducing this information and horizontal springs into the modelling. The frequency ratios in this model, labelled as 'design model' in Fig. 6, are also shown. It can be seen that strengthening the columns influenced the frequencies of horizontal modes strongly ('initial FE model' in Fig. 6), while the added springs then improved correlation with vertical modes ('manually tuned model' in Fig. 6).

Having reduced the maximum frequency error in the manually tuned model to 4.0% and matching the sequence of experimental and FE modes facilitated the successful and physically meaningful automatic updating by the updating software [15]. Also, it can be concluded that very detailed FE modelling and some manual tuning led to a very good correlation between experimental and analytical model. However, it would be interesting to see if/how the automatic updating could improve these results, having in mind that this starting model for automatic updating was much closer to the experimental results than most of the automatically updated models reported in the reviewed literature.

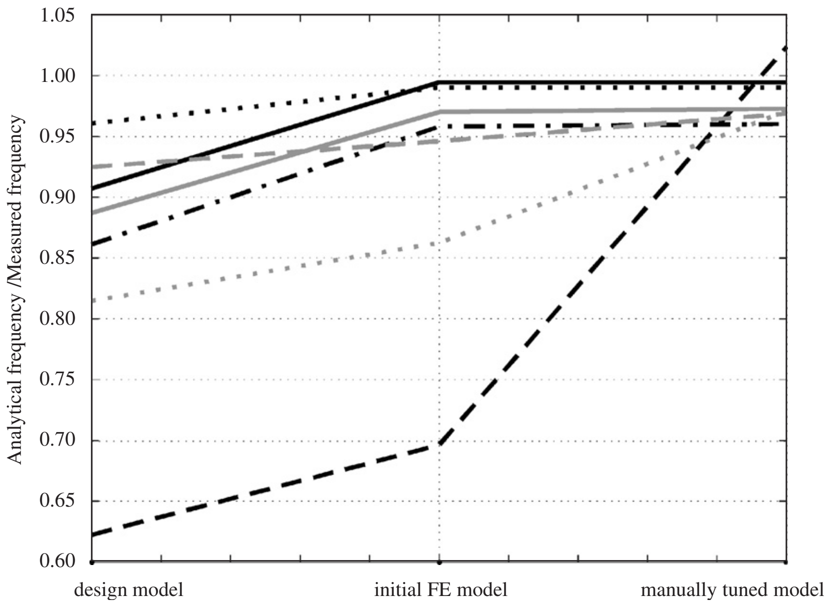

Fig. 6. Manual frequency tuning of different FE models (1HS—solid black line; 1VS—dotted black line; 1VA—dashed black line; 1HA—dot–dashed black line; 2HS—solid grey line; 2VA—dotted grey line; 2VS—dashed grey line).

5. Automatic model updating

The updating procedure was conducted with the aim to improve further the analytical model so that it could be used in more advanced vibration response analysis, which is beyond the scope of this paper. The procedure is based on the theoretical principles outlined in Section 2.2.

5.1. Target response selection

Having in mind the good quality of the experimental data, all seven measured modes of vibration were targeted in the updating process. Both measured natural frequencies and MAC values were taken into account. Therefore, in total 14 target responses were selected for updating. To take into account the lower reliability of identified mode shapes in comparison with measured natural frequencies, the confidence factor for MAC values was ten times lower than that for natural frequencies. This was chosen based on previous experience [8]. These confidence factors feature in the $\mathbf{C}_R$ matrix, part of the error function (Eq. (4)).

5.2. Parameter selection

As previously mentioned, the main criteria for parameter selection are their uncertainty and sensitivity. Therefore, parameters related to the geometry that was not precisely described in the design data available were selected as uncertain. These parameters are shown in Fig. 7. For simplicity, only half of the bridge is presented on the figure having in mind its symmetry with respect to the YZ plane. It can be seen that all parameters that characterise the deck were selected. This is because of uncertain contribution of the asphalt and composite slab to the stiffness of the bridge deck. Besides this, only approximate data about asphalt and concrete thicknesses were available. The same applies to the composite slabs in the column-girder connections (Fig. 2). Also, the fact that the bridge is more than 30-year old may contribute to the deterioration of its components (such as the asphalt layer). Because of the unavailability of precise data related to column strengthening, the thicknesses of the column steel plates as well as their dynamic modulus were also selected for updating. The density of water pipe material was selected to take into account the uncertainty about the amount of water in the pipes. Finally, the stiffnesses of the horizontal–longitudinal support springs at the

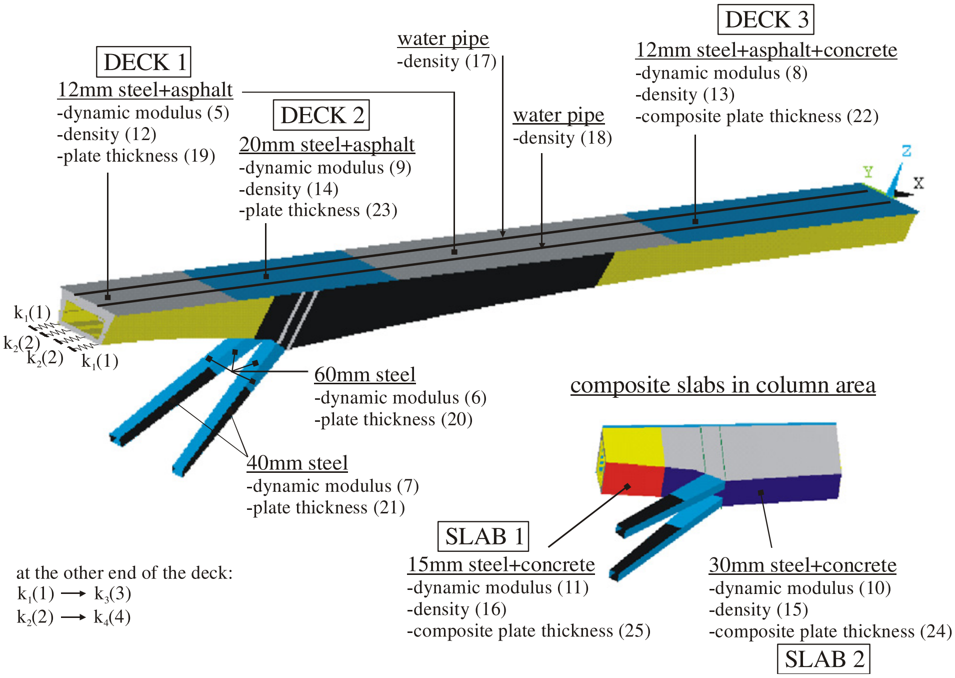

Fig. 7. Uncertain parameters in the manually tuned FE model.

girder ends were also taken into account. In total, 25 parameters were selected, with their sequence number given in parentheses in Fig. 7.

After the parameter selection, a sensitivity analysis was conducted. To be able to compare the sensitivity of different target responses to changes in different parameters, the normalised sensitivity, that is the dimensionless number $S_{n,ij}$, defined as

$$S_{n,ij} = \frac{\Delta R_i / \Delta P_j}{R_i / P_j} = \frac{\Delta R_i}{\Delta P_j} \frac{P_j}{R_i} \tag{9}$$

was calculated for each combination of target responses and parameters. This was done by using the forward finite difference approach [15] with an assumed parameter change of $+1\%$ for all updating parameters.

The plot of sums of normalised sensitivities corresponding to all responses and for all parameters is shown in Fig. 8. It was found that target responses were much less sensitive to three parameters (numbered as 11, 16 and 25 in Figs. 7 and 8) defining a composite slab in the column connection areas in comparison with other parameters. Because of this, these three parameters were excluded from the updating process. All other parameters entered the updating process with their starting values given in Table 3 (column IV). Also, physically meaningful upper and lower limits for these parameters were estimated and are given in columns V and VI.

5.3. Formal updating and its results

The updating procedure was conducted using the FEMtools updating software [15] based on the Bayesian algorithm presented in Section 2.2. The aim was to minimise the error function of the kind defined in Eq. (4), where both natural frequencies and MAC values for mode shapes were selected as target responses. A constraint to the updating procedure was the introduction of the upper and the lower allowable limits for parameter values. The parameter changes per iteration were not limited. For all parameters the same confidence value featuring the matrix $\mathbf{C}_p$ was chosen.

The updating process converged after five iterations. For every mode of vibration the error in calculated natural frequencies compared with their measured counterparts was defined as an absolute value of the

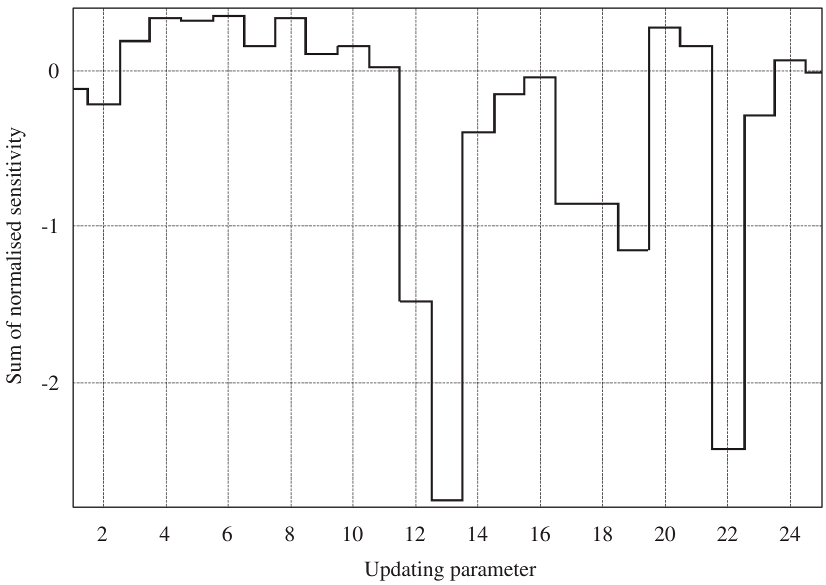

Fig. 8. Plot of sum of normalised sensitivity of 14 selected target responses to 25 uncertain parameters.

relative difference between numerical f_a and experimental f_e natural frequency:

$$\text{frequency error} = \left|\frac{f_a - f_e}{f_e}\right|. \quad (10)$$

The average value of this error across all seven modes for each iteration is presented in Fig. 9. Also, the frequencies and MAC values obtained as a result of the updating process are presented in Table 4. It can be seen that previous maximum frequency difference of 4.0% decreased to 1.2%. Minimum MAC value increased from 0.81 to 0.85, with all other values being well above 0.90. The complete MAC matrix is shown in Fig. 10. The agreement between mode shapes in updated FE model and the experimental data was very good, which can be seen in Fig. 11.

The final parameter values are presented in Table 3 (column VII). The absolute maximum parameter change was 42.6% for the stiffness of a support spring. Only four parameters, amongst 22 selected, reached their allowable limits. The fact that most parameters did not go to their limiting values is a sign of a good parameter choice.

However, when changes in the parameter values through iterations were checked it was found that very large changes occurred in the first iteration. Maximum change was for parameter 2 (k_2) which was −23%. This could be important because of the fact that the Taylor's series given in Eq. (3) was limited to its linear term only. However, the relationship between responses and parameters is, in fact, nonlinear, and having very large changes in parameters in a single iteration can violate the main principles on which the updating procedure was based. Because of this, the updating process was repeated with parameter changes in every iteration limited to 1%—the value which was used to calculate the sensitivity matrix in each iteration. Limiting the maximum parameter change per iteration makes sure that parameter will take new value in the vicinity of the previous value, enabling a more reasonable linear approximation used in Eq. (3). Nevertheless, the new updating setup produced almost the same level of agreement between experimental and numerical target responses as those presented previously. This time, results were obtained after 50 iterations (lasting 10 times longer than previously used five iterations). The agreement of results gave some confidence in their reliability.

Finally, it should be said that an attempt to update the initial FE model (not featuring horizontal springs) under the condition of maximum parameter changes per iteration of 1% led to much worse frequency and MAC correlation (after 140 iterations) although the limits for parameters were free. At the same time, changes

Table 3
The values of starting and updated parameters (E, ρ and h stand for dynamic modulus of elasticity, density and thickness of appropriate elements in FE model, respectively)

I Parameter number (Fig. 7)	II Type	III Structural part	IV Starting value	V Allowed decrease (%)	VI Allowed increase (%)	VII Updated parameter value	VIII Parameter change (%)
1	k_1	Spring support	36.1 (MN/m/m)	No limit	No limit	25.9 (MN/m/m)	−28.6
2	k_2	Spring support	72.1 (MN/m/m)	No limit	No limit	41.4 (MN/m/m)	−42.6
3	k_3	Spring support	36.1 (MN/m/m)	No limit	No limit	30.5 (MN/m/m)	−16.0
4	k_4	Spring support	72.1 (MN/m/m)	No limit	No limit	55.0 (MN/m/m)	−23.7
5	E	Deck 1	210 (GPa)	−10	+10	230 (GPa)	9.5
6	E	Column plate	210 (GPa)	−35	+35	283 (GPa)	35.0
7	E	Column plate	210 (GPa)	−35	+35	228 (GPa)	8.6
8	E	Deck 3	210 (GPa)	−35	+35	141 (GPa)	−32.9
9	E	Deck 2	210 (GPa)	−10	+10	189 (GPa)	−10.0
10	E	Slab 2	210 (GPa)	−35	+35	253 (GPa)	20.5
12	ρ	Deck 1	17475 (kg/m^3)	−20	+20	15987 (kg/m^3)	−8.5
13	ρ	Deck 3	6712 (kg/m^3)	−20	+20	7450 (kg/m^3)	11.0
14	ρ	Deck 2	13625 (kg/m^3)	−20	+20	10900 (kg/m^3)	−20.0
15	ρ	Slab 2	4977 (kg/m^3)	−10	+10	4480 (kg/m^3)	−10.0
17	ρ	Water pipe	4858 (kg/m^3)	−50	+50	4479 (kg/m^3)	−7.8
18	ρ	Water pipe	4858 (kg/m^3)	−50	+50	4475 (kg/m^3)	−7.9
19	h	Deck 1	12 mm	0	+30	13.5 mm	12.5
20	h	Column plate	60 mm	−50	+50	63.9 mm	6.5
21	h	Column plate	40 mm	−50	+50	42.0 mm	5.0
22	h	Deck 3	67 mm	−30	+30	51.2 mm	−23.6
23	h	Deck 2	20 mm	0	+20	20.0 mm	0.0
24	h	Slab 2	110 mm	−20	+20	102.7 mm	−6.6

in some parameters were physically impossible. For example, the thickness of the column plates was about 25 cm which meant that all columns are completely cast in steel, which is obviously wrong. Moreover, the column stiffness was additionally increased via increase in dynamic modulus by a factor of 2. Obviously, the non-existence of the horizontal–longitudinal support springs in the initial FE model required changes in the column parameters which were too large in order to try to correlate vertical modes. Therefore, the manual model tuning conducted before the formal updating proved to be crucial for the success of the formal updating procedure.

6. Discussion

Although a very detailed initial FE model of the Podgorica footbridge was developed based on design data available and best engineering judgement, the discrepancies in the natural frequencies of the first seven modes were quite large between the experimental and numerical results. Particularly poor correlation was obtained for anti-symmetric modes and an error as high as 30% occurred for mode 1VA.

This initial FE model could not be updated in a physically meaningful way by using a sensitivity-based procedure implemented in the FEMtools updating software. This confirmed conclusions found in papers by Pavic et al. [8] and Brownjohn and Xia [9] that the initial FE model usually cannot be updated successfully when large differences between their modal properties and their experimental counterparts exist. This is because these large differences violate the key assumption used in updating that the relationship between response errors and parameter changes in Eq. (3) can be expressed using the first term in the Taylor's series only.

Therefore, the manual tuning which would reconcile as much as practicable the difference between the initial FE model and its experimental counterpart was required before implementing the automatic updating.

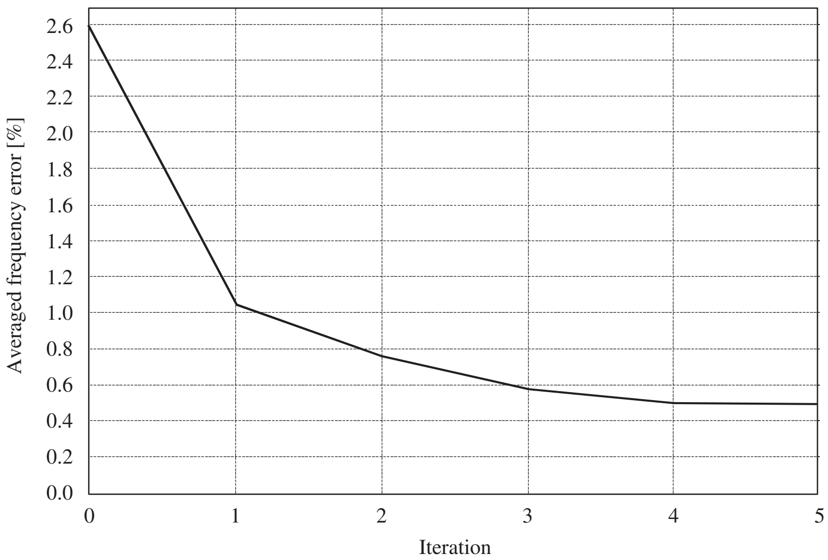

Fig. 9. Convergence of the iterative process presented via averaged frequency error.

Table 4
Correlation between experimental and updated FE model

I Exp. mode #	II Exp. model *f* (Hz)	III Updated model *f* (Hz)	IV Frequency error $(f_{III}-f_{II})/f_{II}$ (%)	V Mode shape correlation MAC (%)
1	1.83 (1HS)	1.84 (1HS)	0.6	99.9
2	2.04 (1VS)	2.05 (1VS)	0.5	99.8
3	3.36 (1VA)	3.38 (1VA)	0.6	99.9
4	4.54 (1HA)	4.50 (1HA)	−0.9	99.3
5	7.35 (2HS)	7.34 (2HS)	−0.1	84.7
6	7.56 (2VA)	7.47 (2VA)	−1.2	93.8
7	7.98 (2VS)	7.98 (2VS)	0.0	98.9

For the bridge investigated, it was found that flexible supports in the longitudinal direction should be introduced instead of free edges at girder ends to improve the correlation between the measured and FE-calculated modes. It seems that the expansion joints at both ends of the bridge deck got jammed due to lack of maintenance and therefore provided a restraint to the bridge movement in the longitudinal direction. Such movement was much more pronounced in the anti-symmetric modes. Also, it might be that the end supports deteriorated and obstructed free movement of the box girder ends.

A simple manual tuning by trial and error guided by engineering judgement was necessary to prepare the FE model for the automatic updating and proved to be crucial for its successful implementation. Of 25 parameters which were considered as uncertain, three were excluded from the updating process because the target responses were not sensitive to them. This is a usual procedure which should help to prevent problems with ill-conditioning during updating. After this, the updating procedure was successfully conducted improving correlation of natural frequencies and MAC values between the final FE and the experimental models. Having said this, it would be interesting here to analyse physical meaning of the parameter changes presented in Table 3.

Water pipes: For both pipes the density approximately decreased for 7.8%. This is an equivalent to the situation when water fills 43% of the pipes volume, a little bit less then the initially assumed 50%.

Deck 1: The stiffness of deck 1 tended to increase through both dynamic modulus of elasticity and thickness of shell elements. This means that the asphalt layer contributed to the overall stiffness of the deck which was neglected when developing the manually tuned FE model. The overall mass of the deck remained approximately the same.

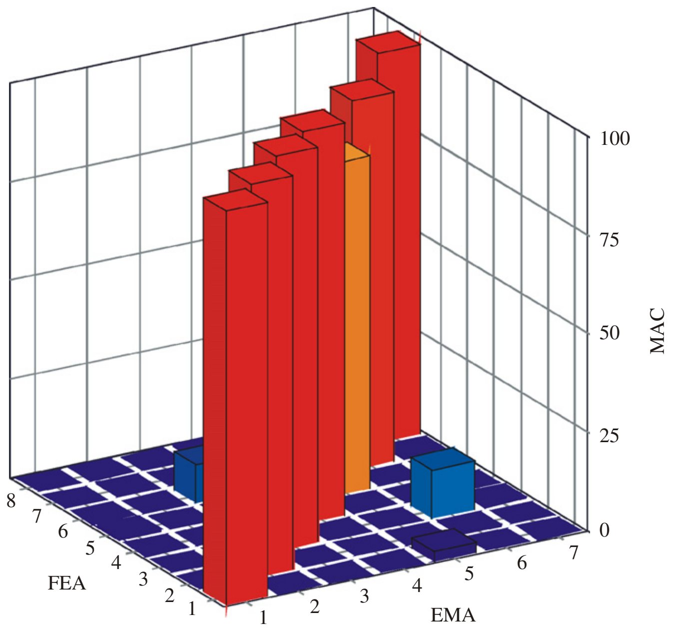

Fig. 10. MAC matrix after updating. FEA and EMA stand for FE model and experimental model, respectively.

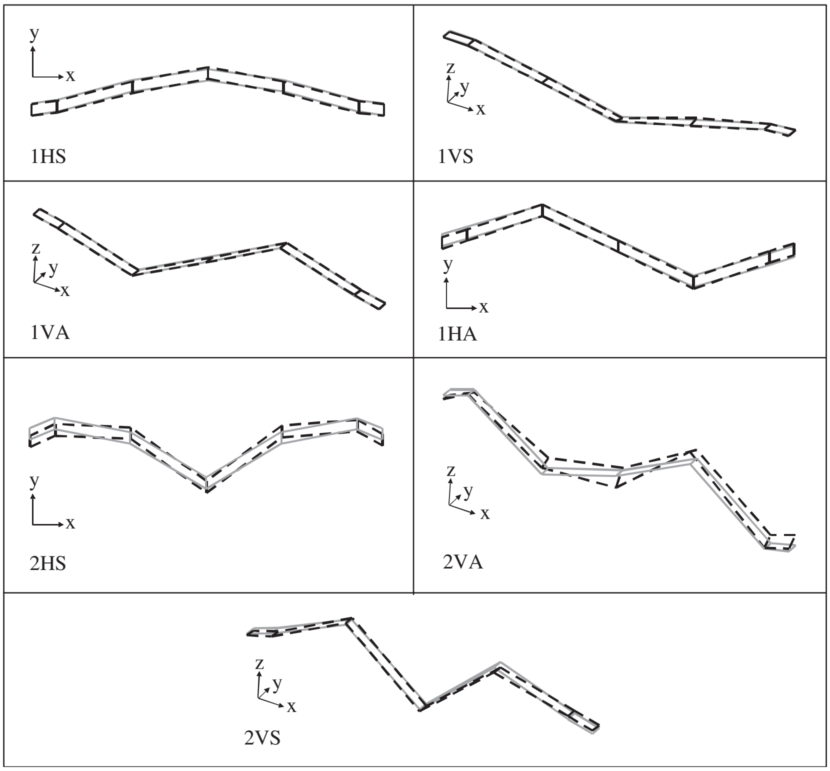

Fig. 11. Overlaying of mode shapes obtained experimentally (black dashed line) and numerically in the final FE model (grey line).

Deck 2: In this area, the shell stiffness tended to decrease as well as the overall mass. Having in mind the asphalt contribution to the stiffness in the deck 1 area, it would be expected that the same happened here but it did not. However, the result obtained for deck 2 probably means that the designed increase in the steel plate thickness from 12 mm in the area of deck 1 to 20 mm in the area of deck 2 was actually not carried out. It was impossible to confirm this information within the scope of this work.

Deck 3: Changes in dynamic modulus of elasticity, shell thickness and its density suggested that the total mass and stiffness of this composite slab are smaller than assumed. This means that the composite slab is composed of 1.2 cm of steel and 7.2 cm of concrete, instead of 1.2 cm of steel and 10.0 cm of concrete, as initially assumed. The 33% decrease in dynamic modulus of elasticity also suggests that the concrete was probably cracked over time and there was possibly certain level of slippage between the steel and concrete layers.

Slab 2: The mass of this composite slab decreased, meaning that the concrete layer is 9.4 cm thick instead of the previously used 13.0 cm. However, the dynamic modulus of elasticity increased by 20%. This, together with the increase of the same parameter for column plates suggests that the whole area of connection between the box girder and columns is very stiff. The exact source of this stiffness is difficult to identify having in mind that there are no precise data about the geometry of columns as well as of the concrete layer in the composite slab. The final plate thicknesses almost stayed unchanged at 6 and 4 cm, which was in agreement with the rule from the national bridge design code in Montenegro. According to this code the plates used for stiffening of a structure can have the thickness which is, as a maximum, the same as the thickness of the original plates.

Longitudinal spring supports: The stiffnesses of these springs were free to increase and decrease. It is interesting here that springs at the right side of the bridge were on average about 25% stiffer than those on the left side. This parameter change probably happened due to an attempt of the numerical procedure to accommodate slight violation of the anti-symmetry in the measured mode 2VA (Fig. 11).

Finally, having in mind that the first vertical mode of vibration is responsible for the footbridge liveliness, the modal parameters related to this mode important for further vibration analysis of the bridge were possible to be identified accurately by combining the FE and experimental results. These are natural frequency of 2.04 Hz (from testing), damping ratio of 0.22% (from testing) and modal mass (from the fully updated FE model) of 58000 kg. This mass was about 10% higher than that obtained in modal testing, being 53188 kg [35].

7. Conclusions

When developing an FE model of the footbridge structure based on the design data available and best engineering judgement where necessary, there is no guarantee that this initial model can reasonably well estimate the modal properties (natural frequencies and mode shapes) of the bridge even when it is very detailed. First seven modes of vibration of the Podgorica footbridge were identified via modal testing. A comparison with their estimates from the initial FE model revealed errors in the natural frequencies, particularly large for two vertical anti-symmetric modes.

An attempt to formally update this design model by changing its input parameters failed producing physically meaningless changes in some parameters. This was due to large differences in the initial and experimental models which cannot be supported by the iterative updating procedure used.

Because of this, a manual tuning of the initial FE model was required with the aim to reconcile these differences. Adding flexible supports to the free edges in the bridge longitudinal direction at the girder ends improved considerably the correlation between the numerical and the experimental models. Only then the numerical model was possible to automatically update via the FEMtools software.

This formal updating further improved the frequency correlation and increased MAC values by changing the values of 22 uncertain and sensitive structural parameters. The fact that all parameter changes were within their physically acceptable limits was very important for judging the updated parameters as meaningful, and therefore the whole of the updating process as successful. The parameter changes suggested that the composite slabs in the bridge were less stiff than assumed. Also, it seemed that the asphalt layer contributed to the deck stiffness.

Acknowledgements

The authors are grateful for the financial support which came from the Universities UK Overseas Research Student Award Scheme (reference ORS/2002036023) and UK Engineering and Physical Sciences Research Council (EPSRC).

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